MA 405-002: Introduction to Linear Algebra and Matrices, NCSU, Spring 2018

Optional Written Homework #6

Due: Thursday, May 3

Please indicate if you would like the grader to give feedback regarding your mathematical writing. This feedback will not be factored into your grade and only serves to help you grow as a mathematician.

If you would like to receive your feedback or your own assignment before the final, please turn it in as early as possible. Solutions will be posted Friday, May 4th and so no late submissions will be accepted.

1. Find the eigenvalues and corresponding eigenspaces of the following matrix. Is A diagonalizable?
$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

2. The following questions refer to $T : P_2 \to P_2$, which is defined by $T(ax^2 + bx + c) = (a - b - c)x^2 + (b - 4c)x + (-2b - c)$.
   (a) Prove T is a linear transformation.
   (b) Find the matrix representation of T with respect to the standard basis of P_2.
   (c) Find the kernel and range of T.
   (d) Is T injective? Is T surjective?
   (e) Find the eigenvalues and eigenvectors of T. (*Note:* Remember that $T : P_2 \to P_2$.)
   (f) Is T diagonalizable? If so, find P and D so that the matrix of T is diagonalizable. (*Note:* Be sure to check your work.)

3. Let $V = M_{2\times 2}$ and $\langle A, B\rangle = \text{trace}(B^T A)$. Show that $(V, \langle A, B\rangle)$ is an inner product space.

4. Let $V = P_2$ with the inner product $\langle p, q\rangle = \int_{-1}^{1} p(x)q(x)dx$. We showed in class that $\mathcal{B} = \{1, x, \frac{1}{2}(3x^2 - 1)\}$ is an orthogonal basis of P_2. Using the inner product, write $f(x) = 1 + x - 2x^2$ in terms of the basis $\mathcal{B}$.
(*Note:* You can check your answer by finding the coordinates of this polynomial with respect to $\mathcal{B}$ as we did before inner products.)