$\iff (A-I)v=0 \qquad A-I=\begin{pmatrix}-1&1\\1&-1\end{pmatrix}\to\begin{pmatrix}1&-1\\0&0\end{pmatrix} \qquad v=\begin{pmatrix}t\\t\end{pmatrix}\ t\in\mathbb{R}$

$\Rightarrow$ The 1-eigenspace is span$\left\{\begin{pmatrix}1\\1\end{pmatrix}\right\}$

$\lambda_2=1$ (on your own) Find v s.t. $Av=(-1)v \iff (A+I)v=0$

$A+I=\begin{pmatrix}1&1\\1&1\end{pmatrix}\to\begin{pmatrix}1&1\\0&0\end{pmatrix} \quad v=\begin{pmatrix}-s\\s\end{pmatrix}\ s\in\mathbb{R}.$

$\Rightarrow$ The (-1)-eigenspace is span$\left\{\begin{pmatrix}-1\\1\end{pmatrix}\right\}$

So Finding eigenvalues is the same as finding what values of λ give the matrix $(A-\lambda I)$ a nontrivial nullspace.

Recap: null$(A-\lambda I)$ nontrivial is the same as...

- corresponding transformation is not injective
- $(A-\lambda I)v=0$ has infinitely many solutions
- rank$(A-\lambda I)<n$
- $A-\lambda I$ is not invertible

So Finding eigenvalues is the same as finding what values of λ give $A-\lambda I_n$ non-invertible?

Useful tool to determine invertibility: Determinants

Recall (or maybe new): We have a simple formula for finding the inverse of a 2x2 matrix:

$A=\begin{pmatrix}a&b\\c&d\end{pmatrix} \qquad A^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}$ and since $ad-bc=0$ is bad!, A is not invertible in that case.

We want to generalize this check to nxn

inductive Def: Let A be an nxn matrix. The determinant of A is the number defined as follows:

① If A is 1x1 $(A=(a))$ then $\det(A)=|A|=a$

② If A is 2x2 $\left(A=\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\right)$ then $\det(A)=|A|=a_{11}a_{22}-a_{12}a_{21}$

Geometrically, $|a_{11}a_{22}-a_{12}a_{21}|$ is area of parallelo. created by $\begin{pmatrix}a_{11}\\a_{12}\end{pmatrix}$ & $\begin{pmatrix}a_{21}\\a_{22}\end{pmatrix}$

③ If A is 3x3, $A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}$ then $\det(A)=a_{11}\begin{vmatrix}a_{22}&a_{23}\\a_{32}&a_{33}\end{vmatrix}-a_{12}\begin{vmatrix}a_{21}&a_{23}\\a_{31}&a_{33}\end{vmatrix}+a_{13}\begin{vmatrix}a_{21}&a_{22}\\a_{31}&a_{32}\end{vmatrix}$

Remember this?

Ex: Find det(A) if

a) $A=\begin{pmatrix}1&2\\3&4\end{pmatrix}$ b) $A=\begin{pmatrix}1&2&0\\-1&3&0\\2&4&1\end{pmatrix}$

Solution:

a) $\begin{vmatrix}1&2\\3&4\end{vmatrix}=1(4)-2(3)=\boxed{-2}$

b) $\begin{vmatrix}1&2&0\\-1&3&0\\2&4&1\end{vmatrix}=1\begin{vmatrix}3&0\\4&1\end{vmatrix}-2\begin{vmatrix}-1&0\\2&1\end{vmatrix}+0\begin{vmatrix}-1&3\\2&4\end{vmatrix}$

$=1(3-0)-2(-1-0)+0$

$=3+2=\boxed{5}$