Then this is the same as asking

"Is $\begin{pmatrix}1\\2\\1\end{pmatrix} \in \text{span}\left\{\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\0\\-1\end{pmatrix},\begin{pmatrix}2\\1\\-1\end{pmatrix}\right\}$?"

That is, is

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 0 & -1 & -1 & 1 \end{array}\right) \text{ consistent?}$$

$$\Big\downarrow \text{RREF}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \text{ consistent ✓ (}\infty\text{ly many solutions)}$$

yes. $\begin{pmatrix}1\\2\\1\end{pmatrix} \in \text{range}(T)$.

$\rightarrow \begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 0 & 0 & 0\end{pmatrix}$

② Using rref of coefficient matrix, we find

$\text{range}(T) = \text{span}\left\{\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\0\\-1\end{pmatrix}\right\}$ So a basis is $\left\{\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\0\\-1\end{pmatrix}\right\}$

$\ker(T)$? We want to say $= \left\{\begin{pmatrix}-s\\-s\\s\end{pmatrix}\right\} = \text{span}\left\{\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right\}$

But, by def, $v \in \ker(T)$ if $T(v)=0 \Rightarrow T(a_1v_1+a_2v_2+a_3v_3)=0$

$\Rightarrow a_1T(v_1)+a_2T(v_2)+a_3T(v_3)=0$

$\Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right)$ and row reduce to find a_i's

Careful! this is actually $a_1+a_3=0$, $a_2+a_3=0$ $\Rightarrow \ker(T) = \text{span}\{-v_1-v_2+v_3\}$

Some general definitions (apply outside of 405) Let A,B be any sets.

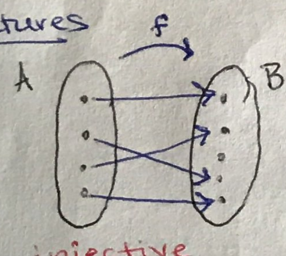
Def: A function $f: A\to B$ is injective if $f(x)=f(y)$ implies $x=y$ } also called one-to-one or 1-1

Equivalently, the contrapositive: if $x\neq y$, then $f(x)\neq f(y)$

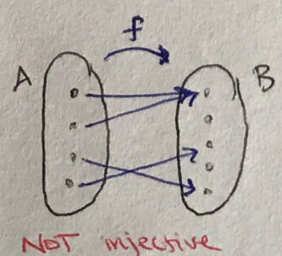
Def: A function $f: A\to B$ is surjective if $\text{range}(f)=B$ (or $f(A)=B$) ← also called onto

Def: If a function is both injective and surjective, then it's called bijective

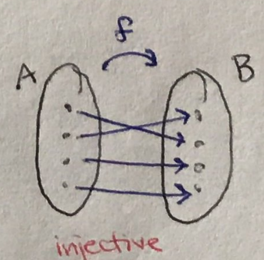
Pictures



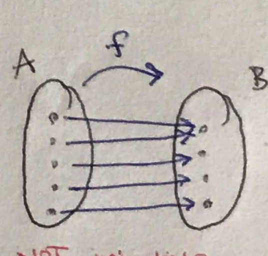
injective
NOT surjective



NOT injective
NOT surjective



injective
surjective
⇒BIJECTIVE



NOT injective
surjective

Ex: $f:\mathbb{R}\to\mathbb{R}$ $f(x)=x^2$

NOT injective: $f(1)=1=f(-1)$ but $1\neq -1$

NOT surjective: $-1\in\mathbb{R}$ (codomain) but $-1\notin \text{range}(f) \Rightarrow \nexists\ x\in\mathbb{R}$ s.t. $f(x)=x^2=-1$

Ex: $g:\mathbb{R}\to\mathbb{R}$ $g(x)=e^x$

Injective: if $e^x=e^y$, then $\ln(e^x)=\ln(e^y) \Rightarrow x=y$.

Not surjective: $\text{Range}(g)=(0,\infty)\neq\mathbb{R}$ (codomain)