# MA 405-002: Introduction to Linear Algebra and Matrices, NCSU, Spring 2018 

## Study Guide for Test \#2

## Basis and Dimension:

What is a basis?

How does this connect to other concepts we've learned in this class?

What is important about a basis?

What does it tell us about the vector space?
Given two bases, $B_{1}$ and $B_{2}$, what can you say about them?

What can you say about the basis of a subspace of a vector space?
Do the ideas of basis and dimension give us anything useful regarding linearly independent or spanning sets?

Practice problems that involve proving a set of vectors is a basis, finding a basis to the span of a set of vectors, finding a basis to a vector space that contains certain vectors, and theoretical proofs involving bases.

Prove the following: $\left\{v_{1}, v_{2}, \ldots, v_{n}\right\} \subset V$ is a basis of $V$ if and only if every $v \in V$ can be written uniquely in the form $v=a_{1} v_{1}+a_{2} v_{2}+\ldots a_{n} v_{n}$ where $a_{1}, a_{2}, \ldots, a_{n} \in F$.

## Systems of Equations:

Given a system of equations, be able to determine the corresponding coefficient and augmented matrices.
What are the row operations we can perform without changing the solution set to a system of equations?

What's Row Echelon Form? Reduced Row Echelon Form? Why do we want our matrices to look like this?

How does RREF relate to linear independence, or other topics we've covered so far?
Practice solving systems of equations. Be able to interpret the solution set from the RREF of a matrix.

## Column Space and Null Space:

What is the rank of a matrix?
How does the rank of a matrix relate to other topics we've learned?

List everything you know about column space and null space below. How do you find bases for each? What are their dimensions? Where do they live? What other questions do they answer?

| Column Space | Null Space |
| :--- | :--- |
|  |  |
|  |  |
|  |  |

## Coordinates and Change of Basis:

What are "coordinates"?

How can coordinates be helpful? How can they be confusing?

What is the change of basis matrix?

How is the change of basis matrix useful? What operation do we need to use it?

Practice switching between representations of a vector. Do you notice anything about your change of basis matrix?

## Linear Maps:

What is a Linear Map?

What are some examples of linear maps? Prove they are linear.

How do linear maps relate to other things we have covered in this class?

What are the properties of linear maps?

Practice proving something is a linear map and finding the matrix representation of a linear map.

