MA 405-002: Introduction to Linear Algebra and Matrices, NCSU, Spring 2018

Written Homework #3

Due: Wednesday, February 28

Use of computer programs to perform Gauss-Jordan elimination is acceptable. Please be sure to state what you used and when you used it.

1. As discussed in class, $B_1 = \{1, x, x^2\}$ and $B_3 = \{1 + x - x^2, 4 + x, 2 - x + x^2\}$ are both bases for P_2.

   (a) Find the change of basis matrix from B_1 to B_3.

   (b) Use what you found in part (a) to confirm that $rep_{B_3}(1 - 2x + 3x^2) = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}_{B_3}$.

2. For each of the following matrices, find bases for the column space and the null space. For each, confirm that the rank-nullity theorem is correct.
   Note: if $\dim(V) = 0$, then $V = \{0\}$. In this case, we do not write a basis (or, we write the empty set).

   (a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 1 \\ -1 & 1 & 2 & 1 & 1 & 0 \\ 0 & 1 & 3 & 2 & 2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 3 & 3 \\ 1 & 2 & 5 & 6 \end{bmatrix}$

3. Find a basis for the span of $S = \{x^3 + x, 2x^3 + 3x, 3x^3 - x - 1, x + 2, x^3 + x^2, x^2 - 8\}$, using vectors from S.

4. Find a basis of $\mathbb{R}^5$ containing the linearly independent set $S = \left\{ \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$.