Review for Test 2

Basis: linearly independent spanning set.
Dimension of vector space = size of basis.
Review ideas & theorems about bases / dimension.

Example Problems:

- Find a basis for the subspace of P_3 defined by $\{p \in P_3 \mid p''(1) - p'(2) - p(3) = 0\}$
- Determine if the set $S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -2 & 1 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix} \right\}$ is a basis for M_2.

Systems of Linear Equations: What row operations do not change the set of solutions? → $R_j \leftrightarrow R_i$, cR_j, $R_j + cR_i$

→ What is our final goal? (REF or RREF)

→ What do the pivots tell us? (# of solutions → in RHS → NO SOLUTION)

↳ about the corresponding column vectors → L.I.!

Example Problems:

- Set up the augmented matrix associated with the following system of equations:

$$\begin{aligned} x_2 - 3x_3 &= 4 \\ x_1 - 2x_2 + x_3 &= 0 \\ -4x_1 + x_2 - x_3 &= 7 \end{aligned}$$

- Perform the first row operation necessary to what you got in order to obtain REF.
- Given the following matrices in RREF, find the solution(s) to the corresponding systems of equations (if they exist).

$$\left(\begin{array}{ccccc|c} 1 & 0 & 4 & 0 & 0 & -1 \\ 0 & 1 & -2 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \left(\begin{array}{ccccc|c} 1 & 0 & 8 & -4 & 0 & 3 \\ 0 & 1 & 2 & -4 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

Column Space and Null Space: → Solution set to homogeneous system of equations for which the matrix is coefficients.

↓

Vector space spanned by the columns

- dim (col space) = rank   dim (null space) = nullity.   rank + nullity = # cols.
- Column Space → RREF and pull the linearly independent columns for basis
- Null Space → free variables in RREF determine relationship among variables. parametrize for basis.

Example Problems:

- Given

$$A = \begin{pmatrix} -1 & 2 & 1 & 4 & 0 \\ -1 & 2 & 1 & -3 & 2 \\ 1 & 1 & 2 & -2 & -1 \end{pmatrix} \quad \text{and } RREF(A) = \begin{pmatrix} 1 & 0 & 1 & 0 & -\frac{10}{7} \\ 0 & 1 & 1 & 0 & -\frac{1}{7} \\ 0 & 0 & 0 & 1 & -\frac{2}{7} \end{pmatrix}$$

Find bases for both the column space and null space of A.

Coordinates and Change of Basis: We learned how to write any vector in any vector space of dimension n as a "vector" in $\mathbb{R}^n$ by creating a coordinate vector using the coefficients when it is written as a linear combination of an ordered basis.

We can easily move between two bases of a vector space by creating the change of basis matrix, & using matrix multiplication

To move <u>from</u> $\mathcal{B}_1 = \{u_1, \dots, u_n\}$ to $\mathcal{B}_2$, we use $\left(\begin{array}{c|c|c|c} | & | & & | \\ rep_{\mathcal{B}_2}(u_1) & rep_{\mathcal{B}_2}(u_2) & \cdots & rep_{\mathcal{B}_2}(u_n) \\ | & | & & | \end{array} \right)$

<u>Example Problems</u>:

- Find $rep_{\mathcal{B}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ where $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$ is a basis for $\mathbb{R}^3$
- Find the CoB matrix from $\mathcal{B}$ to the standard basis for $\mathbb{R}^3$ (and vice versa).
- Multiply the two matrices from ↑ together. What do you get?

<u>Linear maps</u>: <u>functions</u> $T: V \to W$ with $T(u+v) = Tu + Tv$ $\forall u, v \in V$ and $T(\lambda v) = \lambda T v$ $\forall v \in V, \lambda \in F$.

<u>Example Problems</u>:

- Show the map $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \\ -x_2 \end{pmatrix}$ is linear.
- Find $[T]_{\mathcal{B}_1, \mathcal{B}_2}$ where T is defined as above and $\mathcal{B}_1 = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$, $\mathcal{B}_2 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$
- Find $T\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (the image of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ under T).