MA 405-002: Introduction to Linear Algebra and Matrices, NCSU, Spring 2018

Written Homework #1

Due: Friday, January 26

1. Given any $\alpha = a + bi \in \mathbb{C}$ where a and b are not both 0, find c and d so that $c + di = \frac{1}{a+bi}$. [Hint: this is a generalization of what we did in class, though you should think of every possible case].

2. Determine whether or not the set with the given operations is a vector space over $\mathbb{R}$. If it is, prove it. If it is not, say which of the axioms fail to hold.

   (a) $V = \left\{ \begin{pmatrix} t+1 \\ 2t \end{pmatrix} \middle| t \in \mathbb{R} \right\}$ with the usual addition and scalar multiplication.

   (b) $\mathbb{R}^{\mathbb{R}}$ with the addition defined by $f \oplus g = f \circ g$ (composition of functions).

   (c) $P_2 = \{a + bx + cx^2 | a, b, c \in \mathbb{R}\}$, the set of all polynomials of degree less than or equal to 2.

3. Show that in any vector space V over $\mathbb{R}$, if $x, y \in V$ and $a, b \in \mathbb{R}$, then $(a + b)(x + y) = ax + bx + ay + by$.

4. Determine if each of the following subsets of $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$. If it is, prove it. If not, state why not.

   (a) $W = \{(x_1, x_2, x_3) | 4x_1 + 3x_2 - 2x_3 = 0 \text{ and } x_1 - x_3 = 0\}$

   (b) $W = \{(a_1, a_2, a_3) | a_1, a_2, a_3 \in \mathbb{Z}\}$ Note: $\mathbb{Z}$ is the set of all integers.

5. Prove that the set $T = \{p(t) \mid \int_0^2 p(t)dt = 0\} \subset P_2$ is a vector space. This proof should be similar to those we did in class, using the proposition about subspaces.

6. Show that the subset $W = \left\{ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in M_2(\mathbb{R}) \middle| a_{11} = a_{22} = 0 \text{ and } a_{12} = -a_{21} \right\}$ (called the set of *skew-symmetric* 2 by 2 matrices) is a subspace of $M_2(\mathbb{R})$.