Ex What is span(S) where $S=\left\{\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}1\\1\\1\end{pmatrix},\begin{pmatrix}1\\0\\0\end{pmatrix}\right\}$ from earlier? You showed $\mathbb{R}^3$.

So is $\text{span}\left(\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right)$ ← building blocks for $\mathbb{R}^3$, can have more than one spanning set.

Ex $S=\{1, x, x^2\}$ is a spanning set for P_2.

Ex Define $P=\{a_0+a_1x+a_2x^2+\dots+a_nx^n \mid a_i\in F,\ n$ any natural number$\}$ $n\in\mathbb{Z}_{>0}$

What is this?

→ set of all polynomials

What are my building blocks?

→ $P=\text{span}(1, x, x^2, \dots, x^n, \dots)$

Def: A vector space is finite dimensional if it has a finite spanning set. If it does not have a finite spanning set, it is an infinite dimensional vector space.

Ex Describe $\text{span}\left(\begin{pmatrix}5&0\\0&0\end{pmatrix},\begin{pmatrix}0&-3\\-3&0\end{pmatrix},\begin{pmatrix}0&0\\0&2\end{pmatrix}\right)$ in a different way.

Apply the definition of span → $a\begin{pmatrix}5&0\\0&0\end{pmatrix}+b\begin{pmatrix}0&-3\\-3&0\end{pmatrix}+c\begin{pmatrix}0&0\\0&2\end{pmatrix}=\begin{pmatrix}5a&-3b\\-3b&2c\end{pmatrix}=\begin{pmatrix}k_1&k_2\\k_2&k_3\end{pmatrix}$

any constant

This is the set of all symmetric 2x2 matrices

⇒ vector space since by Thm, it's a subspace of $M_2(\mathbb{R})$

Ex $\text{span}\left(\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\3\\0\end{pmatrix}\right)=\text{span}\left(\begin{pmatrix}0\\1\\0\end{pmatrix}\right)=\left\{\begin{pmatrix}0\\c\\0\end{pmatrix} \mid c\in\mathbb{R}\right\}$

NOTE: We can't always "just replace with 1" for example: $\text{span}\left(\begin{pmatrix}2\\3\\0\end{pmatrix}\right)$ ← ratio is important

$=\text{span}\left(\begin{pmatrix}1\\\frac{3}{2}\\0\end{pmatrix}\right) \neq \text{span}\left(\begin{pmatrix}1\\1\\0\end{pmatrix}\right)$

Thought... $S=\left\{\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\3\\0\end{pmatrix}\right\}$ was too big. We found a more efficient set to describe the set. How do we detect this?

Ex Do the following sets span P_2?

$S_1=\{1+x, x^2\}$ $\qquad S_2=\{x^2, 1+x, 1\}$ $\qquad S_3=\{x^2, x, 1, 2x^2-x\}$

↓

using just linear combinations of $1+x$ & x^2, can we get any polynomial of degree ≤ 2?

i.e. Does $P_2=\text{span}(S_1)$?

$\text{Span}(S_1)=\{a(1+x)+bx^2 \mid a,b\in\mathbb{R}\}$

elements look like bx^2+ax+a → can't get constants (for example)

No! $\text{span}(S_1)\subsetneq P_2$ (it's a "proper" subspace)

← Notation: "is contained in, but not equal to" also $\subset$

S_2? Can we write $ax^2+bx+c=\underline{a}\,x^2+\underline{b}\,(1+x)+\underline{c-b}\,1$

Yes!

S_3? $ax^2+bx+c=\underline{a}\,x^2+\underline{b}\,x+\underline{c}\,1+\underline{0}\,(2x^2-x)$

Yes! S_3 spans P_2, but it is bigger than we needed

What do we mean by "bigger?"