MA405-002 Introduction to Linear Algebra

Linear Algebra - what is it?

Linear maps / transformations

Big Picture: Study of Vector Spaces and functions between them

Objects: Vectors, linear equations + systems of linear equations, Matrices

SUPER USEFUL: Calculus, differential equations, engineering, computer science, chemistry, physics, statistics, economics PROJECT!

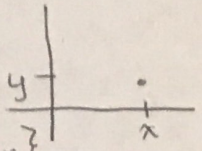
Vector Spaces

Begin with some examples...

Ex1 $\mathbb{R}^2 = \{(x,y) \mid x,y \in \mathbb{R}\}$

How can we think of elements of $\mathbb{R}^2$? - ordered pairs, points on x-y plane, vector from origin to (x,y), vector, ordered lists of length 2

Notation: (x,y), $\langle x,y \rangle$, $\begin{pmatrix} x \\ y \end{pmatrix}$, $x \in \mathbb{R}^2$, $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$



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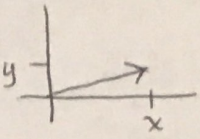

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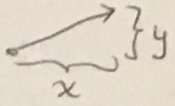
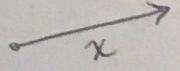


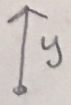
$\mathbb{R}^3, \mathbb{R}^4, \ldots \mathbb{R}^n = \{(x_1, \ldots, x_n) \mid x_i \in \mathbb{R} \ \forall i = 1, \ldots, n\}$

What can we do in $\mathbb{R}^n$? • add vectors → commutative, associative, has identity (0) & inverse

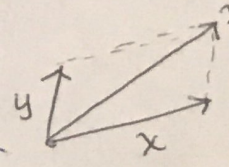
• multiply vector by a scalar → distributive, associative

Ex2 More abstractly,





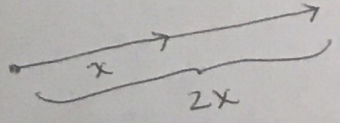
if these vectors are forces both acting on same object:



object will end up here.

This is the same as adding two points in $\mathbb{R}^2$: $(4,1) + (1,2) = (5,3)$

OR we could apply twice the amount of force in the x direction:



→ Same as multiplying a scalar to a point in $\mathbb{R}^2$:

$(4,1) \cdot 2 = (8,2)$

Examples for you: • In $\mathbb{R}^3$, $x = (2,4,5)$ $y = (-4,3,1)$. Find $x - 4y$

$$\begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} - 4\begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 18 \\ -8 \\ 1 \end{pmatrix}$$

• Find $x \in \mathbb{R}^4$ so that $(6,-4,3,5) + x = (7,-2,6,9)$ $\begin{pmatrix} 6 \\ -4 \\ 3 \\ 5 \end{pmatrix} + \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \\ 6 \\ 9 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$

New Ex3 $\mathbb{C} = \{a+bi \mid a,b \in \mathbb{R}, i^2 = -1\}$

$5 \in \mathbb{C}$ $(5+0i)$, $7i \in \mathbb{C}$ $(0+7i)$, $-1+4i \in \mathbb{C}$

We can add & multiply elements of $\mathbb{C}$: $(-1+4i) + (2+3i) = 1+7i$

$(-1+4i)(2+3i) = -2+8i-3i+12i^2 = -14+5i$

For you.

Ex: Find $\alpha \in \mathbb{C}$ so that $\alpha(2+3i) = 1$. Hint: $(a+bi)(2+3i) = 1$

$2a+2bi+3ai-3b = 1$

$2a-3b = 1$

$2b+3a = 0 \Rightarrow b = -\frac{3a}{2}$ $\Rightarrow 2a + \frac{9a}{2} = 1$

$\frac{13a}{2} = 1 \Rightarrow a = \frac{2}{13} \Rightarrow b = \frac{-3\left(\frac{2}{13}\right)}{2} = -\frac{3}{13}$

★ HW! ★

$$\alpha = \frac{2}{13} - \frac{3}{13}i$$